Electric field

Electric field is the electric force per unit charge.

The one charge exert force on another charge by means of disturbances that are generated in space surrounding the charges. These disturbances are electric fields.

* Fields endowed with energy and momentum.

Mathematically it is represented as

$$\vec{E} = \frac{\vec{F}}{Q}$$

Let there be two charges q_1 and q_2

Electric field generated by point charge q_1

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}$$

The force that this electric field exerts on the charge q_2 $\vec{F} = q_2 \vec{E}$

The direction of electric field is radially outward if q_1 is positive and radially inward if q_1 is negative.

As we have discussed earlier in Principle of Superposition, force exerted on q due to number of charges $q_1, q_2, q_3 \ldots$ is written as

$$\vec{F} = \frac{1}{4\pi\epsilon_0} q \sum_{i=1}^{N} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

So, Electric field of source charges will be

$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

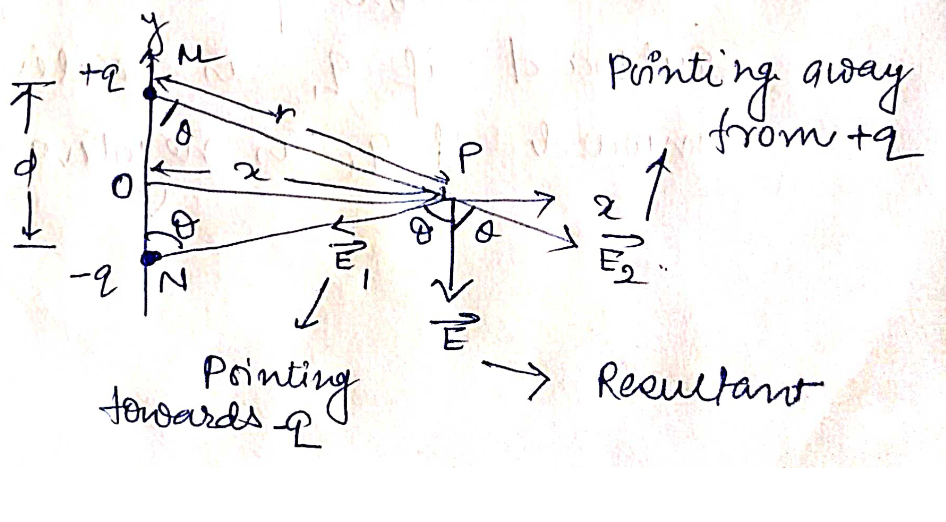
Net electric field generated by any distribution of point charges can be calculated by forming the vector sum of the individual electric fields.

* S.I unit of electric field is Newton per Coulomb (N/C) or volts per meter.

\# Electric field intensity at a point is called as field strength at that point.

Problem: Consider two charges $\pm q$ of equal magnitude and opposite signs, separated by a distance d. ~~Such an arrangement of charges~~, Find the electric field at a point equidistant from two charges, a distance x from their mid points

Solution:



The magnitude of each of the two Individual electric field

$$E_1 \text{ (due to } -q) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E_2 \text{ (due to } +q) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

r is distance between point P and charge q and $-q$.

$$\text{so, } r = \sqrt{x^2 + (d/2)^2} = \sqrt{x^2 + d^2/4}$$

At point P, the electric field due to +q charge points away ~~from~~ and electric field due to the $-q$ charge points towards the charge.

x component of E_1 = $E_1 \sin\theta$ (upward)

x component of E_2 = $E_2 \sin\theta$ (downward)

Net x component of electric field $E_x = E_1 \sin\theta - E_2 \sin\theta$

$$E_x = 0$$

Net y component of electric field E_y

$$E_y = \underbrace{-E_2 \cos\theta}_{\text{Pointing in negative direction}} \underbrace{- E_1 \cos\theta}_{\text{Pointing in negative direction}}$$

$\therefore \quad E_y = -2E_1 \cos\theta$

as $E_1 = E_2$

$$\therefore \quad E_y = -2 \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \cos\theta$$

$$= -2 \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + d^2/4)} \cos\theta$$

$$\cos\theta = \frac{b}{h} = \frac{d/2}{\sqrt{x^2 + d^2/4}}$$

From Δ MOP

$$\cos\theta = \frac{d/2}{\sqrt{x^2 + d^2/4}}$$

[As base $= d/2$
hypotenuse $= r$]
$r = \sqrt{x^2 + d^2/4}$

$$E_y = -2 \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + d^2/4)} \frac{d/2}{(x^2 + d^2/4)^{1/2}}$$

$$= -\frac{qd}{4\pi\varepsilon_0 (x^2 + d^2/4)^{3/2}}$$

Here negative sign showing the direction of electric field in "negative y direction".

Magnitude of electric field

so, $E = \sqrt{E_x^2 + E_y^2} = \frac{1}{4\pi\varepsilon_0} \frac{qd}{(x^2 + d^2/4)^{3/2}}$

when $x >> d$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{dq}{x^3}$$